SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2009

YEAR 11 ASSESSMENT TASK #1

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- Each question is to be returned in a separate booklet.
- All necessary working should be shown in every question.
- Answer in simplest exact form unless otherwise stated.
- Full marks may not be awarded for careless or badly arranged work.

Total Marks – 80

- Attempt questions 1 – 5
- All questions are of equal value.

Examiner: Ms F Nesbitt

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

Question 1 (16 marks)

(a) Solve $|x-2|<3$ and graph your answer on a number line **2**

(b) Rationalise the denominator and simplify **2**

$$\frac{3+2\sqrt{5}}{2\sqrt{5}-1}$$

(c) Differentiate and simplify: **5**

(i) $\dfrac{x^2}{2}-3\sqrt{x}$

(ii) $\dfrac{6x+5}{1-3x}$

(iii) $\sqrt{2x+8}$

(d) Solve the following equation. $\log_2(x+4)-\log_2(x-2)=1$ **2**

(e) A parabola has equation $(x-3)^2=8y$ **2**

Find: (i) the coordinates of its vertex

(ii) the equation of its axis of symmetry

(f) Find all the values of m for which the following quadratic equation has real roots: **3**

$$mx^2-8x+m=0.$$

Question 2 (16 marks) **<u>Start a new booklet</u>**

(a) The function $f(x)$ is defined by the rule: **3**

$$f(x) = 0 \ \text{if} \ x \le 0$$
$$f(x) = 2x \ \text{if} \ x > 0$$

(i) Sketch the function in the Domain $-2 \le x \le 2$

(ii) Find the area between $f(x)$ and the x axis.

(b) For the function whose derivative is **2**

$$\frac{dy}{dx} = x^2(3x-1)(x-2),$$

determine the nature of the turning point at the point where $x=0$

(c) Solve: $3^{x-5} = 7$ correct to 2 decimal places. **2**

(d) Given that there is a root of $kx^2 - 20x + k = 0$ at $x = 3$, **3**

find the value of the other root.

(e) Find, from first principles, the derivative of $x^2 - 3$. **3**

(f) On a diagram, mark clearly the region for which **3**

$y \ge -\sqrt{1-x^2}, y \ge -x$ *and* $y \le 0$ are true simultaneously.

Question 3 (16 marks) **<u>Start a new booklet</u>**

(a) Find $\lim_{x\to 1}\dfrac{x^2-x}{x^2-1}$ **2**

(b) If the roots of the equation $x^2-5x+2=0$ are α *and* β, 4

Find the values of

(i) $\alpha+\beta$

(ii) $\alpha\beta$

(iii) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$

(c) Solve: **5**

(i) $x^2-8x+10=0$

(ii) $9^x-4(3)^x+3=0$

(d) Find the domain over which the graph of the function **3**

$y=\dfrac{2}{3}x^3-\dfrac{5}{2}x^2-3x$ is concave up

(e) The probability that a train will be late on any given day is $\dfrac{1}{5}$. **2**

Find the probability that, over a 3 day period, the train will be:

(i) late each day

(ii) late at least once.

Question 4 (16 marks) **<u>Start a new booklet</u>**

(a) The graph of $y = f(x)$ passes through the point $(3,1)$ and **3**

$\frac{dy}{dx} = 1 + \frac{3}{x^2}$. Find $f(x)$

(b) For the curve $y = x^3 - 3x^2$ **9**

(i) Find any stationary point(s)

(ii) Determine the nature of the stationary point(s)

(iii) Find any point(s) of inflexion

(iv) Sketch the curve in the domain $-1 \le x \le 3$ showing all the above features.

(c) A parabola has equation $x = 7 + 6y - y^2$ 4

Find (i) the coordinates of its vertex,

(ii) its focal length,

(iii) the equation of its directrix.

Question 5 (16 marks) **<u>Start a new booklet</u>**

(a) Solve $12 \times 8^{x-2} = \dfrac{3}{4^x}$ **2**

(b) Differentiate $\dfrac{1}{\sqrt{x-1}-\sqrt{x}}$ **3**

You are not required to rationalise the denominator in your answer.

(c) If α and β are the roots of $x^2 - 8x + 5 = 0$, **3**

find a quadratic equation with roots α^2 and β^2

(d) Find the equation of the tangent and normal to the curve 4

$y = x^3 - x^2$ at the point $(2,4)$

(e) The cost of running a long distance truck is **4**

$\left(\dfrac{1}{3}v^2 + 200\right)$ dollars per hour where v is the speed in km/h.

(i) Show that the cost for k kilometers is

$\dfrac{k}{v}\left(\dfrac{1}{3}v^2 + 200\right)$ dollars.

(ii) Find the value of v which will minimise the cost.

End of the Paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}dx = \ln x,\ x > 0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln\left(x + \sqrt{x^2 - a^2}\right)\ \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

Q1 (a) $|x-2| < 3$

$\therefore -3 < x-2 < 3$

$\therefore -1 < x < 5$

2

(b) $\dfrac{3+2\sqrt{5}}{2\sqrt{5}-1} \times \dfrac{2\sqrt{5}+1}{2\sqrt{5}+1}$

$= \dfrac{6\sqrt{5}+3+20+2\sqrt{5}}{20-1}$

$= \dfrac{23+8\sqrt{5}}{19}$ 2

(c) (i) $\dfrac{d}{dx}\left(\dfrac{x^2}{2} - 3\sqrt{x}\right)$

$= x - \dfrac{3}{2}x^{-\frac{1}{2}}$ 1

$= x - \dfrac{3}{2\sqrt{x}}$

(ii) $\dfrac{d}{dx}\left(\dfrac{6x+5}{1-3x}\right)$

$= \dfrac{(1-3x).6 - (6x+5)\times -3}{(1-3x)^2}$

$= \dfrac{6-18x+18x+15}{(1-3x)^2}$ 2

$= \dfrac{21}{(1-3x)^2}$

(iii) $\dfrac{d}{dx}\left(\sqrt{2x+8}\right)$

$= \dfrac{1}{2}(2x+8)^{-\frac{1}{2}}.2$ 2

$= \dfrac{1}{\sqrt{2x+8}}$

(d) $\log_2(x+4) - \log_2(x-2) = 1$

$\log_2 \dfrac{x+4}{x-2} = 1$

$\therefore \dfrac{x+4}{x-2} = 2$

$\therefore x+4 = 2x-4$ 2

$x = 8$

(e) $(x-3)^2 = 8y \quad = 4\times 2y$

(i) Vertex is $(3,0)$ 1

(ii) Axis of symmetry is $x=3$ 1

(f) For real roots

$\Delta \geq 0$

$64 - 4\times m\times m \geq 0$

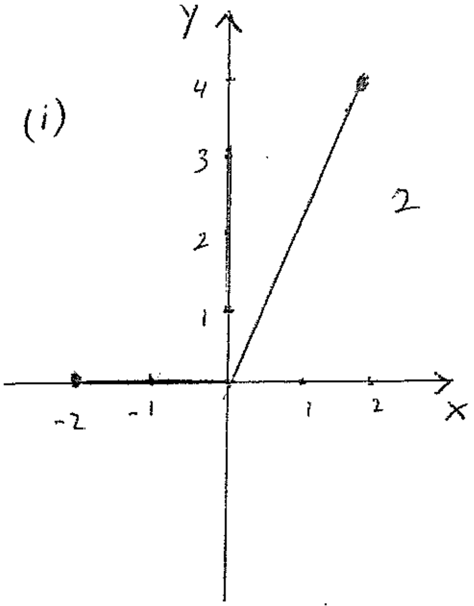
$64 - 4m^2 \geq 0$

$\therefore 4m^2 \leq 64$

$m^2 \leq 16$ 3

$\therefore -4 \leq m \leq 4$

2 (a) (i)



(ii) $\text{Area} = \frac{1}{2} \times 2 \times 4$ 1

$= 4 \ u^2$

2

(b) at 0, $\frac{dy}{dx} = 0$; it is positive at $x = 0^-$ and $x = 0^+$

It is a horizontal point of inflexion. 2

(c) $3^{x-5} = 7 \quad \therefore \quad x - 5 = \frac{\ln 7}{\ln 3} \quad \therefore \quad x = 6.77$ 2

(d) Product of roots $= \frac{c}{a} = \frac{K}{K} = 1$. Other root $= \alpha$.

$\therefore \quad 3\alpha = 1 \quad \therefore \quad \alpha = \frac{1}{3}$ 3

(e) $f(x) = x^2 - 3$

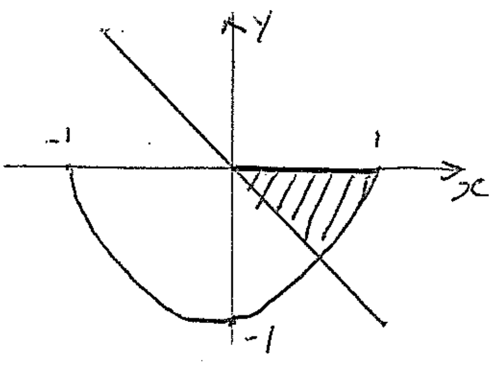
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left(\frac{x^2 + 2xh + h^2 - 3 - (x^2 - 3)}{h} \right) \quad 3$$

$$= \lim_{h \to 0} \left(\frac{2xh}{h} + \frac{h^2}{h} \right)$$

$$= 2x$$

(f)



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QUESTION 3

a) $\lim_{x\to 1} \dfrac{x^2-x}{x^2-1}$

$= \lim_{x\to 1} \dfrac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$

$= \lim_{x\to 1} \dfrac{x}{x+1} = \dfrac{1}{2}$

b) $x^2-5x+2=0$.

$(x-\alpha)(x-\beta)=0$.

i) $\alpha+\beta = -b/a = 5$.

ii) $\alpha\beta = c/a = 2$

iii) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha} = \dfrac{\alpha^2+\beta^2}{\alpha\beta}$.

$= \dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$.

$= \dfrac{5^2-2(2)}{2}$

$= \dfrac{21}{2}$.

c)(i) $x^2-8x+10=0$.

$x^2-8x+10+6=6$.

$(x-4)^2=6$.

$x=4\pm\sqrt{6}$.

(ii) $9^x-4(3^x)+3=0$.

$(3^x)^2-4(3^x)+3=0$.

$(3^x-3)(3^x-1)=0$.

$3^x=3$ or $3^x=1$.

$x=1$ or $x=0$

d) $y=\frac{2}{3}x^3-\frac{5}{2}x^2-3x$

is concave up when $y''>0$.

$y'=2x^2-5x-3$.

$y''=4x-5$

where $y''>0$

$4x-5>0$.

$x>5/4$.

concave up. domain is $x\in\mathbb{R}: x>5/4$

e) $P(\text{late})=\frac{1}{5}$.

P(late and late and late)

$= \frac{1}{5}\times\frac{1}{5}\times\frac{1}{5} =$

$= 1/125$.

P(late at least once)

$= 1 - P(\text{not late} \times 3)$

$= 1-(4/5)^3$

$= 61/125$.

Solutions to Q(4).

Q(4)

(a) $\dfrac{dy}{dx} = 1 + 3x^{-2}$. (3)

$y = x - 3x^{-1} + c$ 2

When $x = 3$, $y = 1$

$1 = 3 - \frac{3}{3} + c$ 1

$\therefore 1 = 2 + c \Rightarrow c = -1$

$\therefore y = x - \frac{3}{x} - 1$.

(b) $y = x^3 - 3x^2$

(i) $\dfrac{dy}{dx} = 3x^2 - 6x$

$= 3x(x-2)$

(ii) $\dfrac{dy}{dx} = 0$, when $x = 0$,

$(0,0)$ $(2,-4)$. 3.

$f''(x) = 6x - 6$ 2.

$f''(0) = -6 < 0$, max

$f''(2) = 6 > 0$, min

(iii) $f''(x) = 0$ (9)

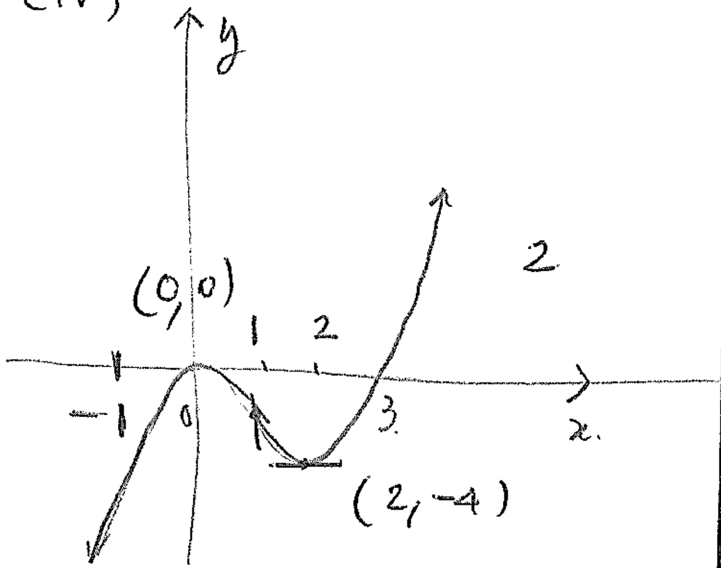
$x = 1$, $y = -2$.

x	$\frac{1}{2}$	1	2
$f''(x)$	-3	0	6

2

$\therefore (1,-2)$ is a pt of inflexion.

(iv)



2

$-y^2 + 6y + 7 = x$. (4)

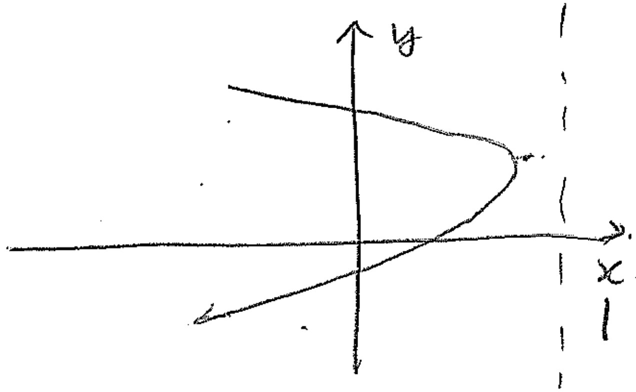
$y^2 - 6y - 7 = -x$

$(y-3)^2 - 16 = -x$.

$(y-3)^2 = -(x - 16)$ 2

$\therefore$ Vertex $(16, 3)$.

$a = \frac{1}{4}$ 1



1

$x = 16\frac{1}{4}$

$x = \frac{65}{4}$

Q5.

(a) $3 \times 2^2 \times 2^{3x-6} = 3 \times 2^{-2x}$.

$2^{3x-4} = 2^{-2x}$.

$3x - 4 = -2x$

$5x = 4$

$x = \frac{4}{5}$.

(b) $y = \dfrac{1}{\sqrt{x-1} - \sqrt{x}} \times \dfrac{\sqrt{x-1} + \sqrt{x}}{\sqrt{x-1} + \sqrt{x}}$.

$= \dfrac{\sqrt{x-1} + \sqrt{x}}{x - 1 - x}$.

$y = -\sqrt{x-1} - \sqrt{x}$.

$\dfrac{dy}{dx} = -\frac{1}{2}(x-1)^{-\frac{1}{2}} - \frac{1}{2}(x)^{-\frac{1}{2}}$

$= -\dfrac{1}{2\sqrt{x-1}} - \dfrac{1}{2\sqrt{x}}$.

(c) Method 1.

$X = x^2 \Rightarrow \sqrt{X} = x.$

$\sqrt{X}^2 - 8\sqrt{X} + 5 = 0.$

$X + 5 = 8\sqrt{X}.$

$(X+5)^2 = 64X.$

$X^2 + 10X + 25 = 64X.$

$X^2 - 54X + 25 = 0.$

Method 2.

$\alpha + \beta = 8$

$\alpha\beta = 5.$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta.$

$= 64 - 10$

$= 54 = -\frac{b}{a}.$

$\alpha^2\beta^2 = (\alpha\beta)^2$

$= 25 = \frac{c}{a}.$

let $a = 1$

$x^2 - 54x + 25 = 0$

(ii) $C = \frac{k}{3}v + 200kv^{-1}$

$\frac{dC}{dv} = \frac{k}{3} - 200kv^{-2}$

$\frac{k}{3} - 200kv^{-2} = 0.$

$v^2 = 600$

$v = \pm 10\sqrt{6}.$

Taking the positive velocity.

$v = 10\sqrt{6}$

$\frac{d^2C}{dv^2} = 400kv^{-3}.$

at $v = 10\sqrt{6}$

$\frac{d^2C}{dv^2} = \frac{400k}{(10\sqrt{6})^3} > 0.$

minima.

$\therefore$ velocity that minimises cost is

$10\sqrt{6}$ km/hr.